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1. Angular Momentum operators ($\hat{L}$) or $\hat{L}_x, \hat{L}_y, \hat{L}_z$

classically, angular momentum is given by the vector product of position ($\vec{r}$) and linear momentum ($\vec{p}$) as:

$$\vec{L} = \vec{r} \times \vec{p}$$

if $\vec{i}$, $\vec{j}$ and $\vec{k}$ are unit vectors along x, y and z co-ordinates, we have

$$\vec{r} = \vec{i}x + \vec{j}y + \vec{k}z \quad \text{and}$$

$$\vec{p} = \vec{i}p_x + \vec{j}p_y + \vec{k}p_z$$

Thus,

$$\vec{L} = (\vec{i}x + \vec{j}y + \vec{k}z) \times (\vec{i}p_x + \vec{j}p_y + \vec{k}p_z)$$

$$\text{or, } \vec{L} = \vec{i}(yp_z - zp_y) + \vec{j}(zp_x - xp_z) + \vec{k}(xp_y - yp_x) \quad \text{--- (1)}$$

Also, by definition

$$\vec{L} = \vec{i}L_x + \vec{j}L_y + \vec{k}L_z \quad \text{--- (2)}$$

from eqn (1) and (2), we get

$$\hat{L}_x = yp_z - zp_y = \frac{h}{2\pi i} \left(y \frac{\partial}{\partial z} - z \frac{\partial}{\partial y} \right)$$

$$\hat{L}_y = zp_x - xp_z = \frac{h}{2\pi i} \left(z \frac{\partial}{\partial x} - x \frac{\partial}{\partial z} \right)$$

$$\hat{L}_z = xp_y - yp_x = \frac{h}{2\pi i} \left(x \frac{\partial}{\partial y} - y \frac{\partial}{\partial x} \right)$$

②

2. Linear Momentum operator ($\hat{P}_x$)

for an electron wave, the wave function is represented by

$$\psi(x) = Ae^{\pm \frac{2\pi i x}{\lambda}}$$

Differentiating the above eqn. with respect to x , we get

$$\frac{\partial \psi(x)}{\partial x} = \pm \frac{2\pi i}{\lambda} (Ae^{\pm \frac{2\pi i x}{\lambda}}) = \pm \frac{2\pi i}{\lambda} \psi(x)$$

from de-Broglie equation, we have

$$\lambda = \frac{h}{P_x}$$

Thus,

$$\frac{d\psi(x)}{dx} = \pm \frac{2\pi i}{h} P_x \psi(x)$$

$$\text{or, } P_x \psi(x) = \pm \frac{h}{2\pi i} \frac{\partial \psi(x)}{\partial x}$$

Removing $\psi(x)$ from both the sides, we get the linear momentum operator as

$$\hat{P}_x = \pm \frac{h}{2\pi i} \frac{\partial}{\partial x}$$

The operator in + direction is $\frac{h}{2\pi i} \frac{\partial}{\partial x}$

The operator in - direction is $-\frac{h}{2\pi i} \frac{\partial}{\partial x}$